LON-CAPA Mathematical Functionality

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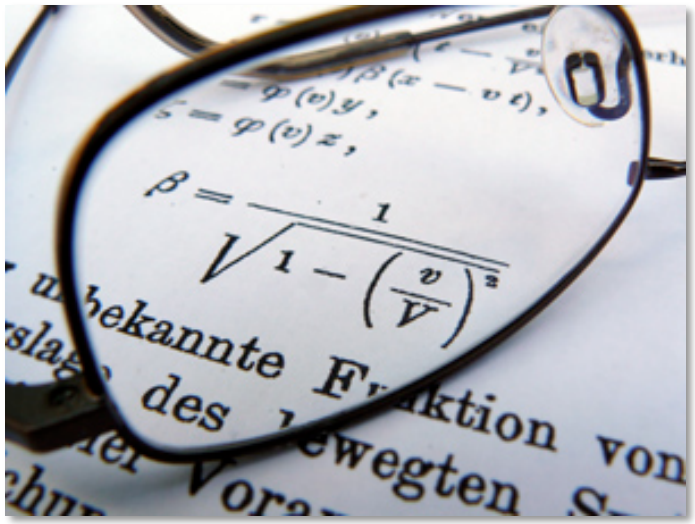
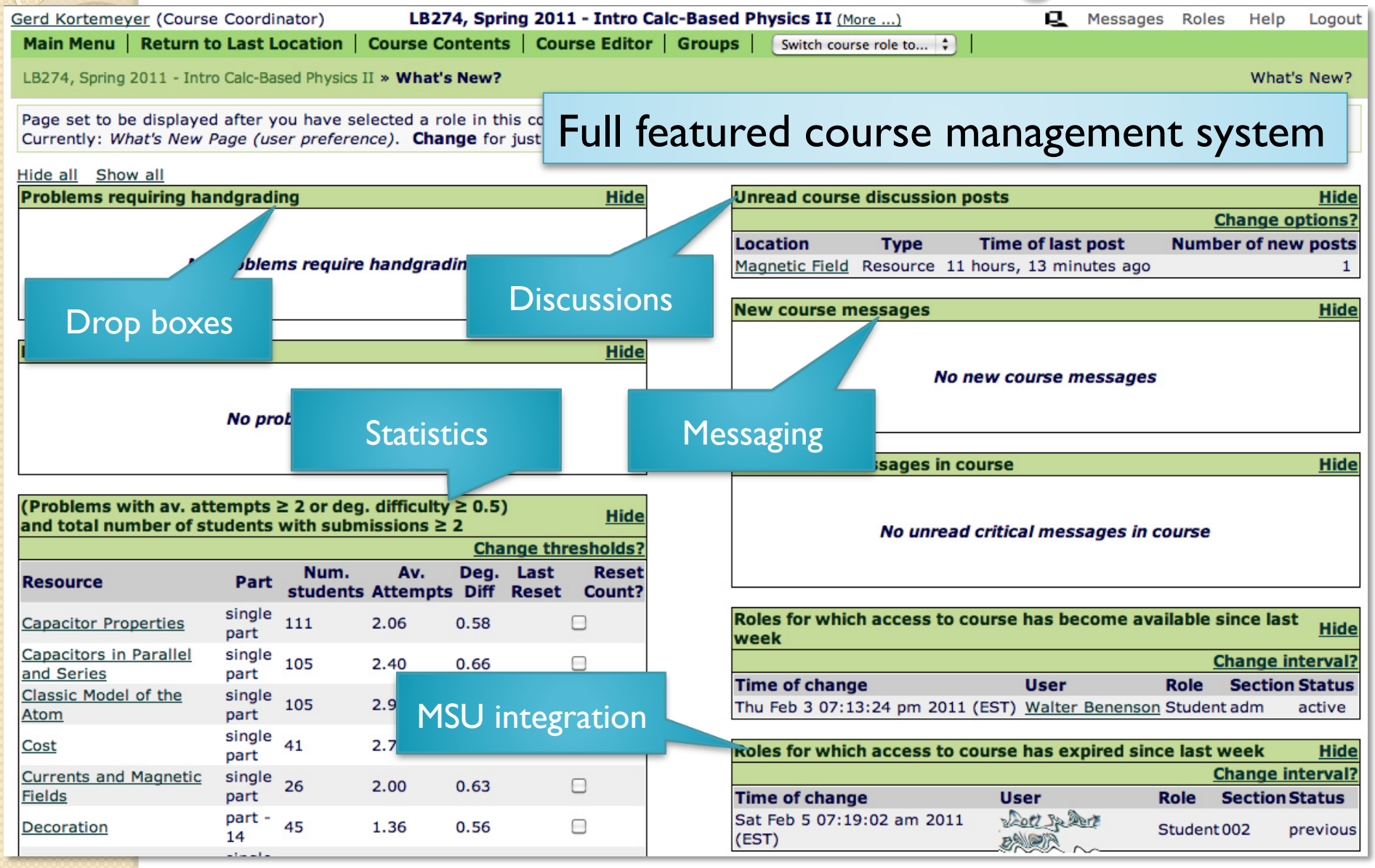
Explorations in Instructional Technology
Brownbag Seminar
February 2011

LON-CAPA Math Functionality

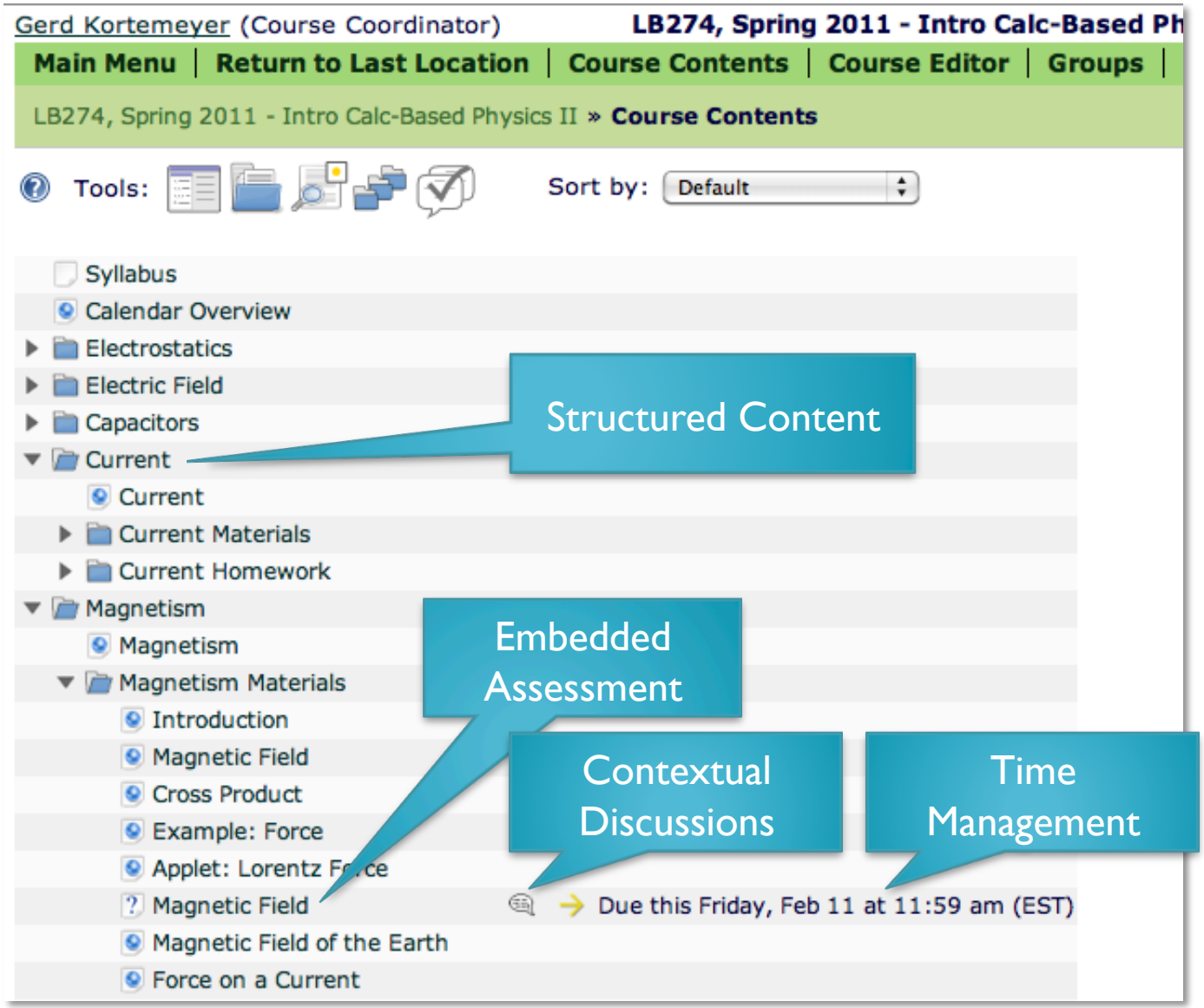


- Today's Session: somewhat specialized topic
- Goal: demonstrate how mathematical assessment is integrated into the LON-CAPA course management functionality

LON-CAPA Course Management

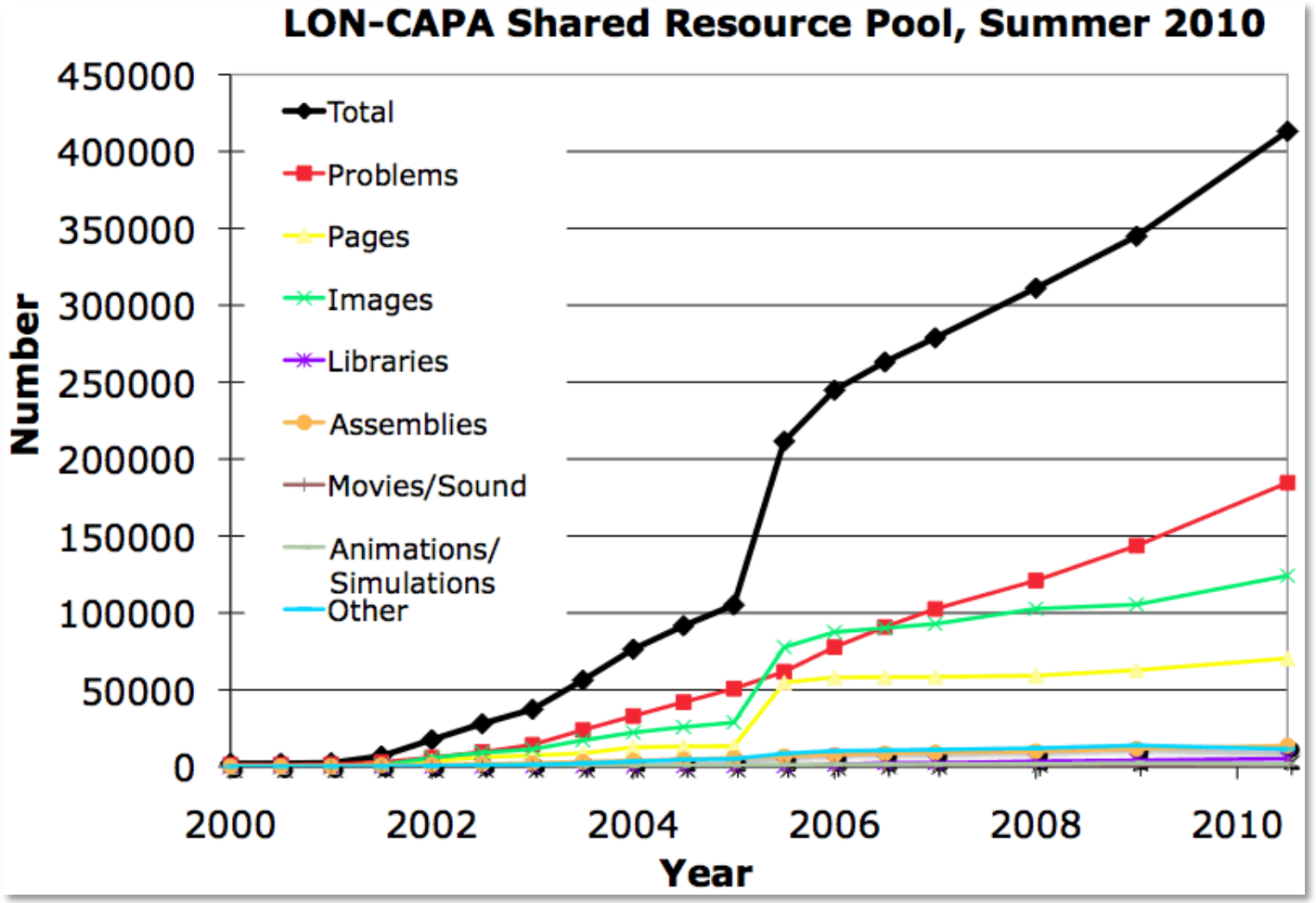


LON-CAPA Course Management



LON-CAPA Course Management

- Shared Learning Content Management



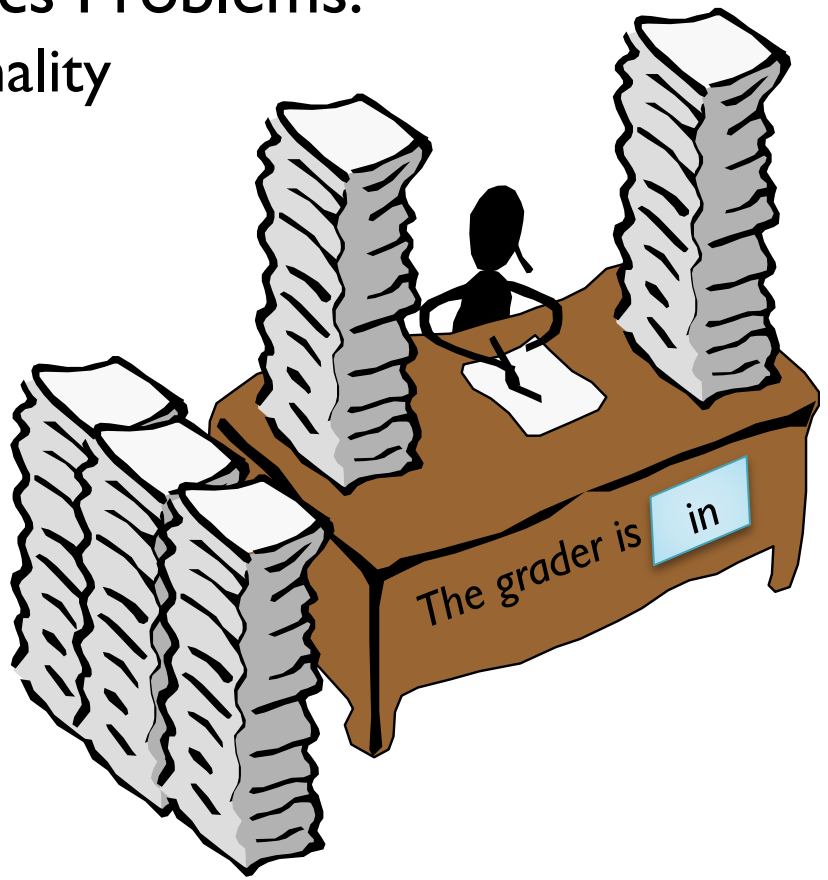
Particular Strength: Assessment

- Randomized problems: different
  - numbers
  - formulas
  - graphs
  - images
  - options
  - …

  for each student.
- Student can collaborate without "cheating"
- Randomized exams

LON-CAPA Mathematics

- Mathematics Output:
  - typesetting
  - graphing
- Generating Mathematics Problems:
  - symbolic math functionality
  - statistics packages
- Mathematics Input:
  - numerical
  - formula evaluation
    - sampling
    - symbolically
    - checking for properties
  - graphical input
  - bubble sheets
  - clickers

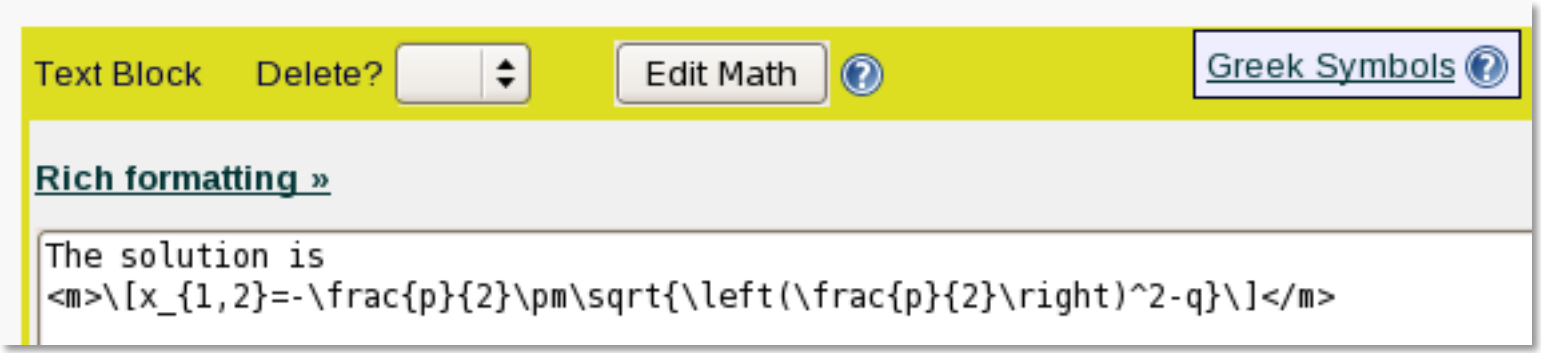


WARNING

Today's presentation is going to show some very specialized functionality. Because you can does not mean you have to.

Mathematical Output

- Typesetting:
  LaTeX can be embedded anywhere in the material

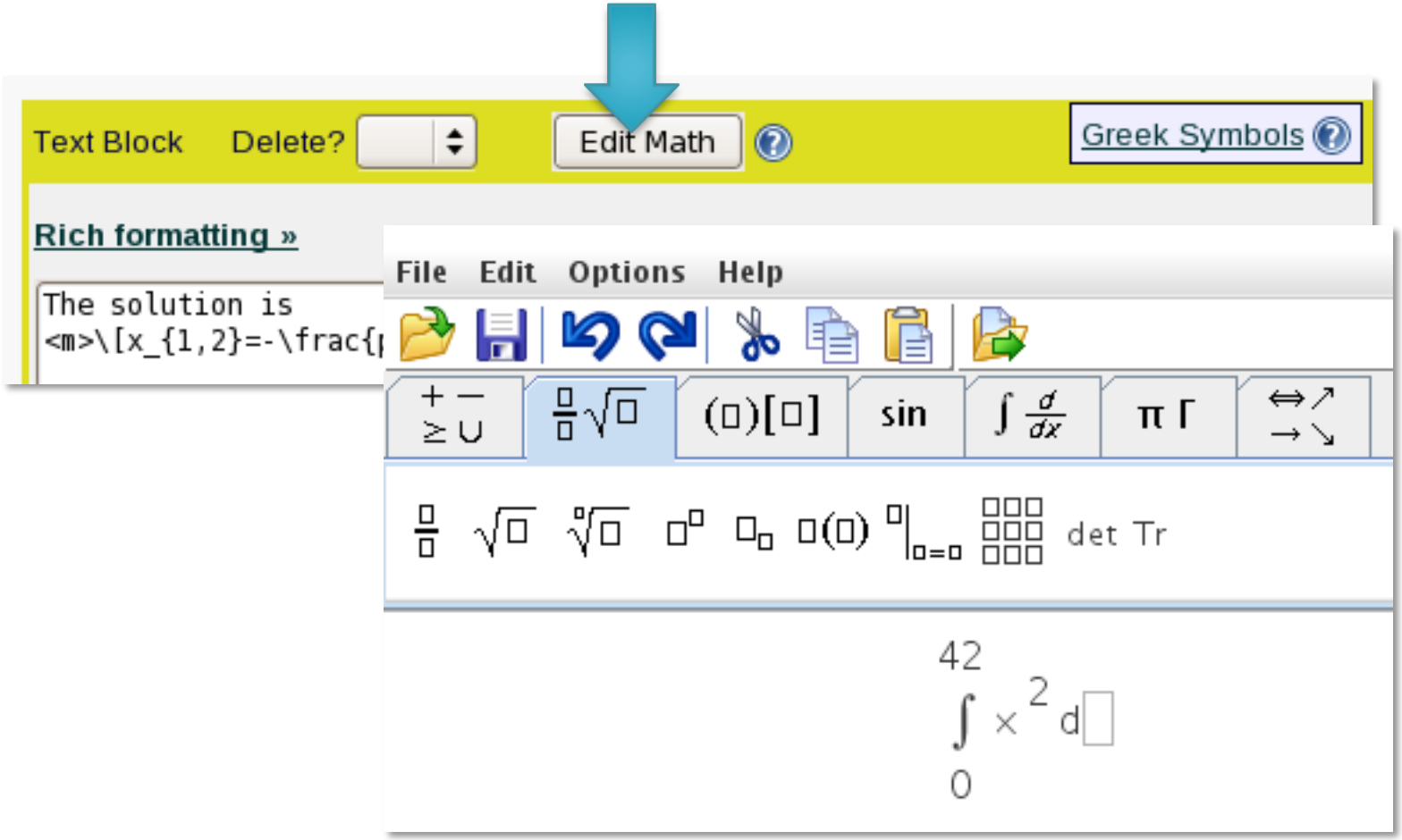


The solution is

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

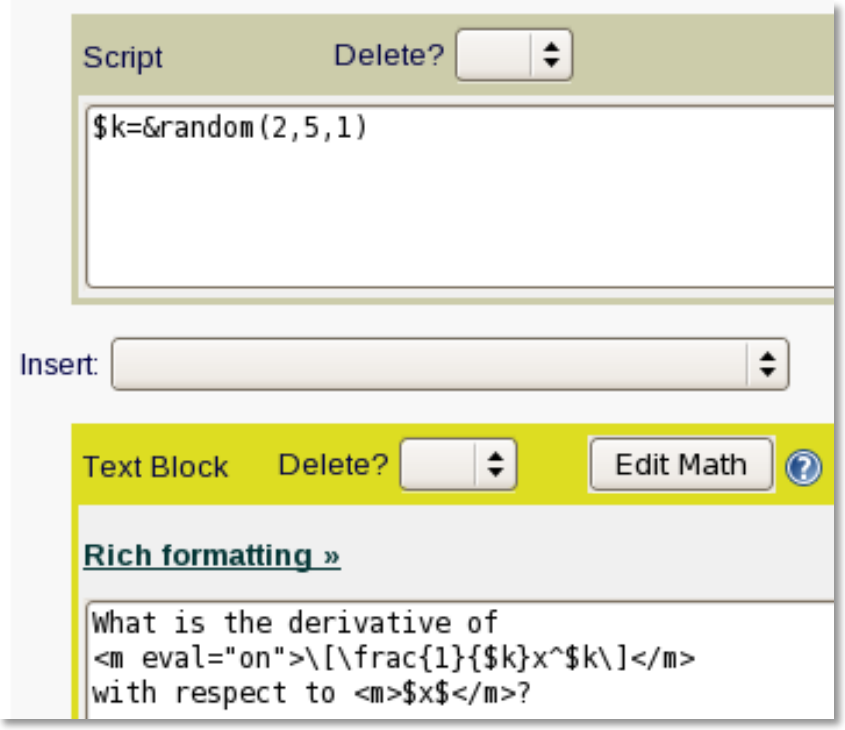
Mathematical Output

- Editor for non-native LaTeX speakers



Mathematical Output

- Generated on-the-fly, can vary from student to student.

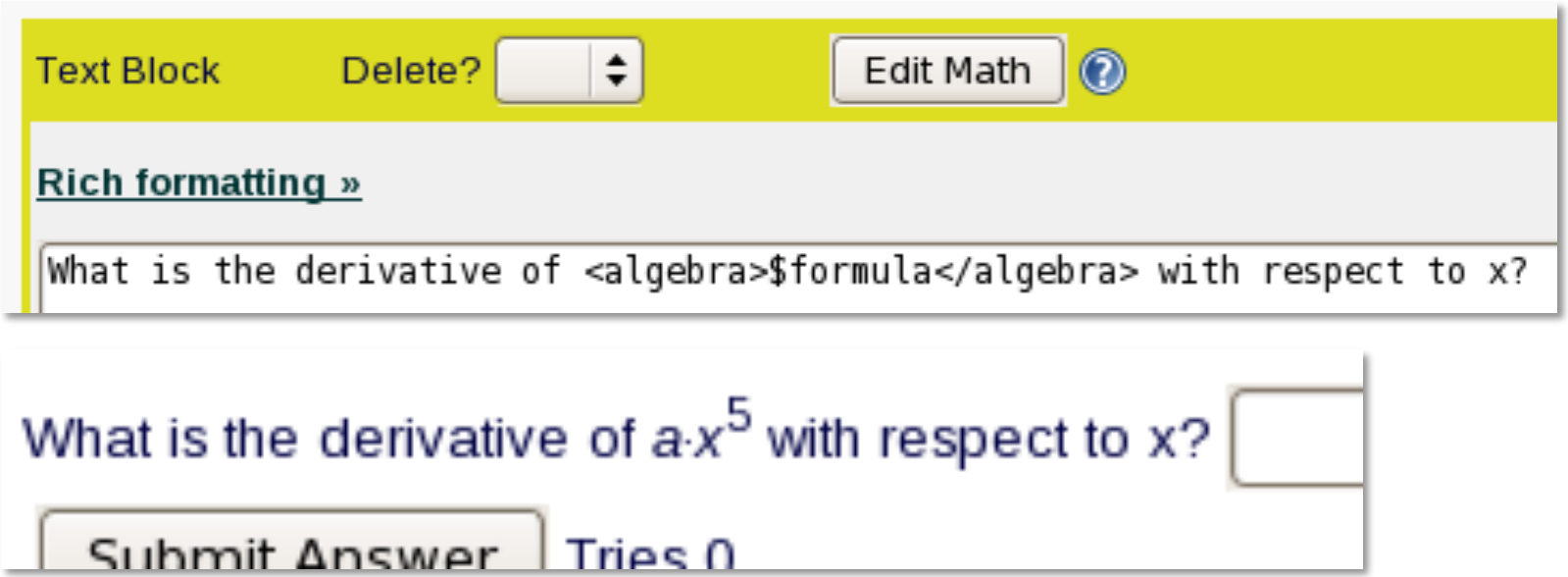


What is the derivative of $\frac{1}{3}x^3$ with respect to x?

What is the derivative of $\frac{1}{5}x^5$ with respect to x?

Mathematical Output

- <algebra>-tag to pretty-print the output from computer algebra systems
- Example: `$formula="a*x^5"`



Mathematical Output

- One-source, multiple target
- Looks good in print
  - Online:

The solution is

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

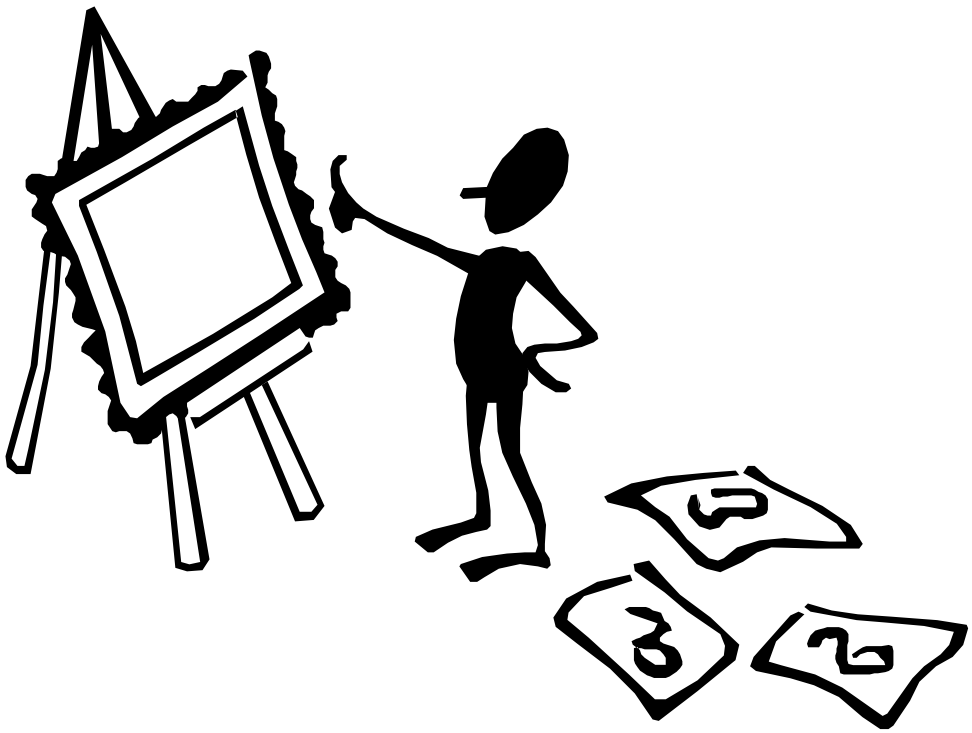
  - Print (dynamically generated PDF):

The solution is

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

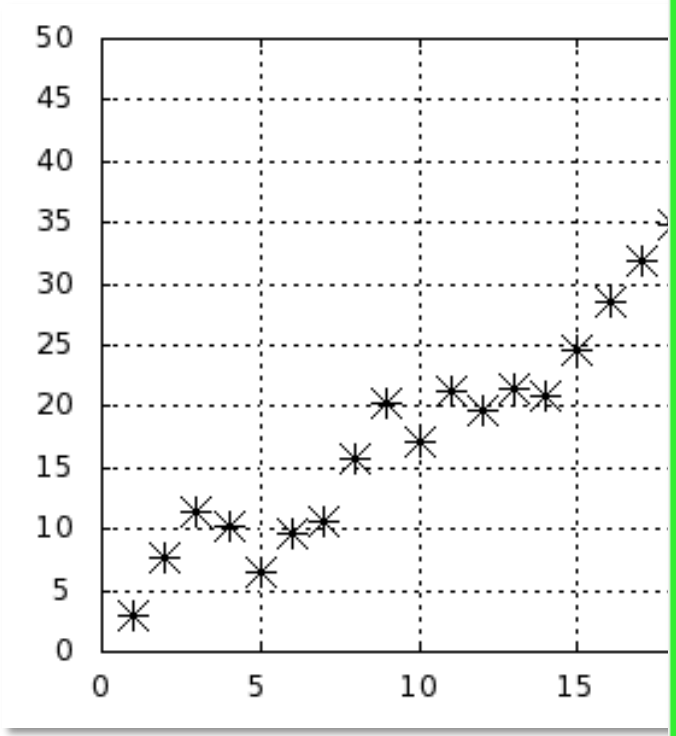
Mathematical Output

- Dynamic Graphing
  - Data-Points
  - Functions
  - Line-Graphics
- Internally uses GNUplot



Mathematical Output

- Data points



Curve Delete?

Color of curve (x000000) x000000

Name of curve to appear in key

Plot with: points

Line width (may not apply to all plot styles) 1

Line type (may not apply to all plot styles) solid

Point type (may not apply to all plot styles) 3

Point size (may not apply to all plot styles) 2

Point to fill -- for filledcurves closed

Comma or space deliminated curve data

@x

Insert:

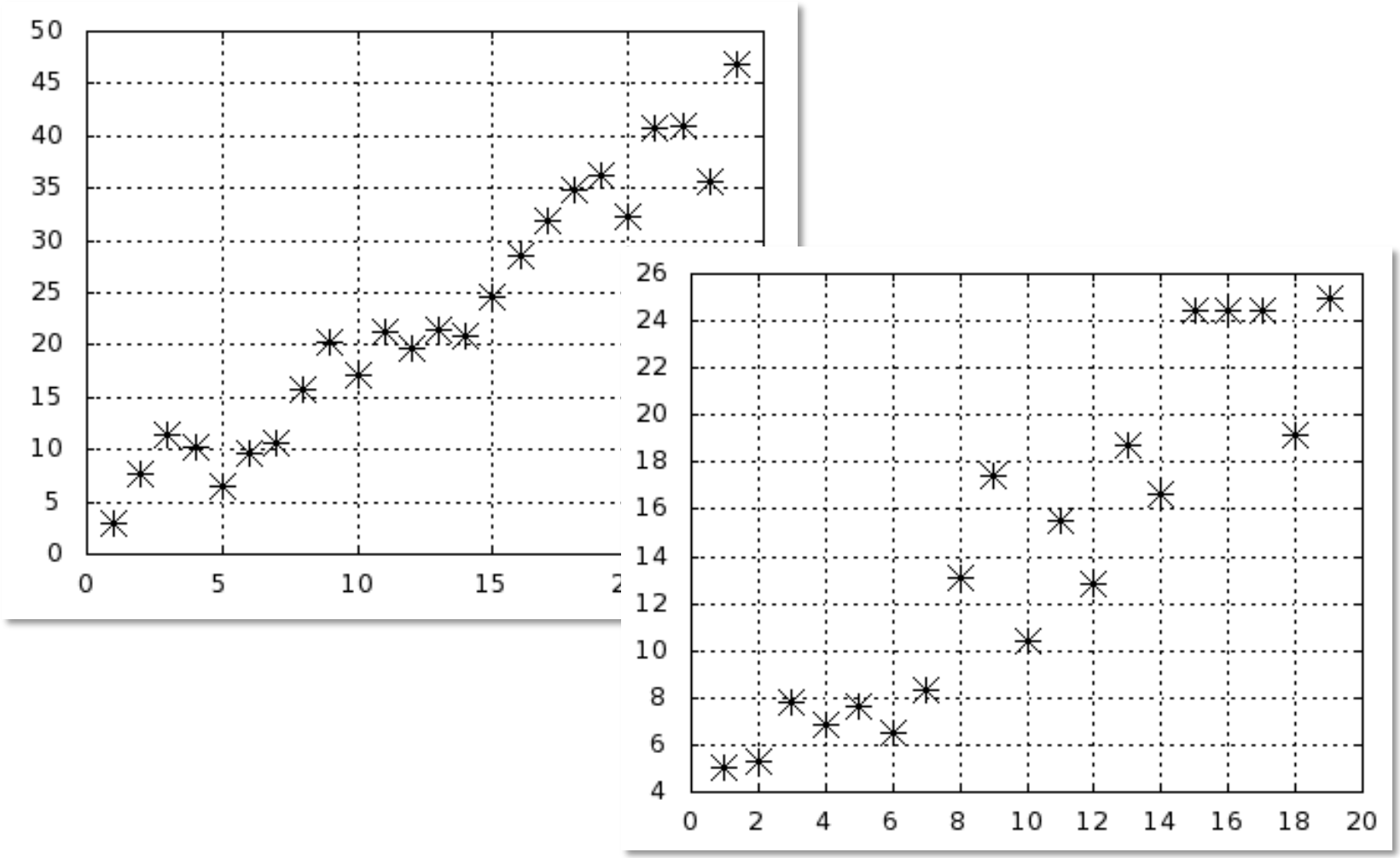
Comma or space deliminated curve data

@y

Insert:

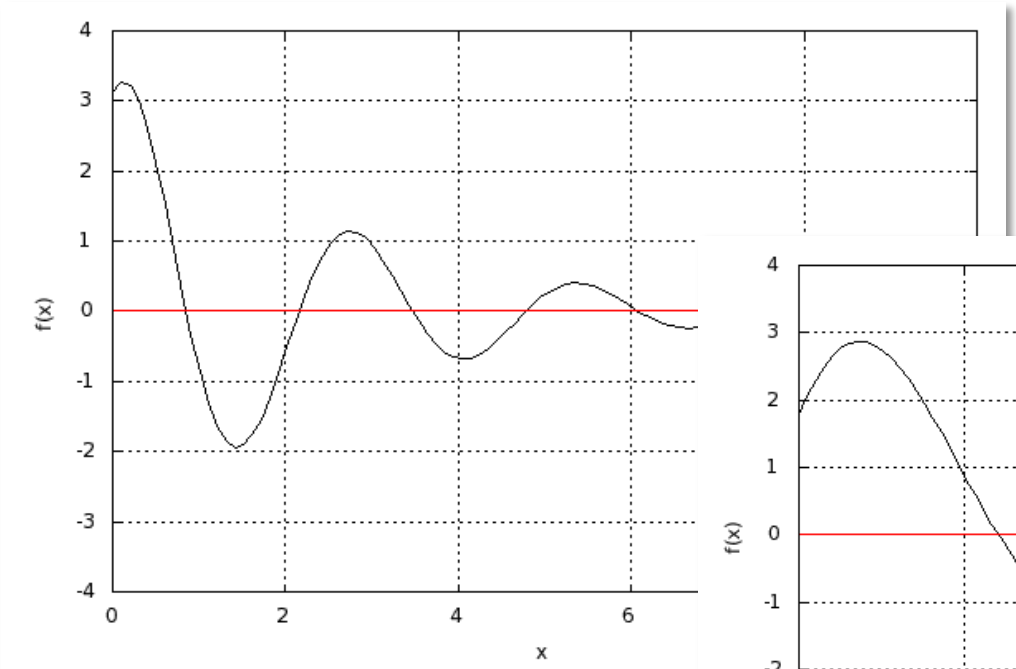
Mathematical Output

- Data points



Mathematical Output

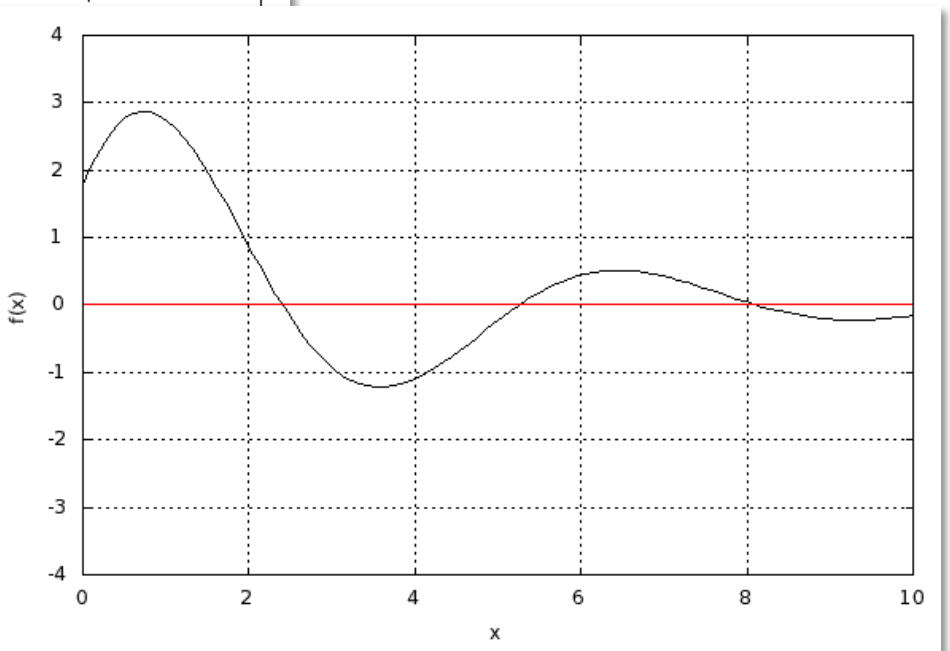
- Functions



Match the function indicated in black.

f(x)=

Submit Answer Tries 0/99



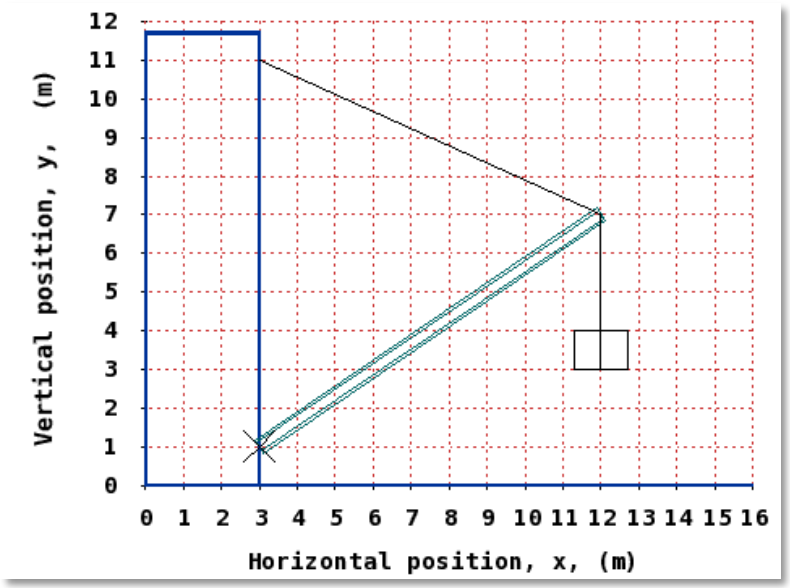
Match the function indicated in black.

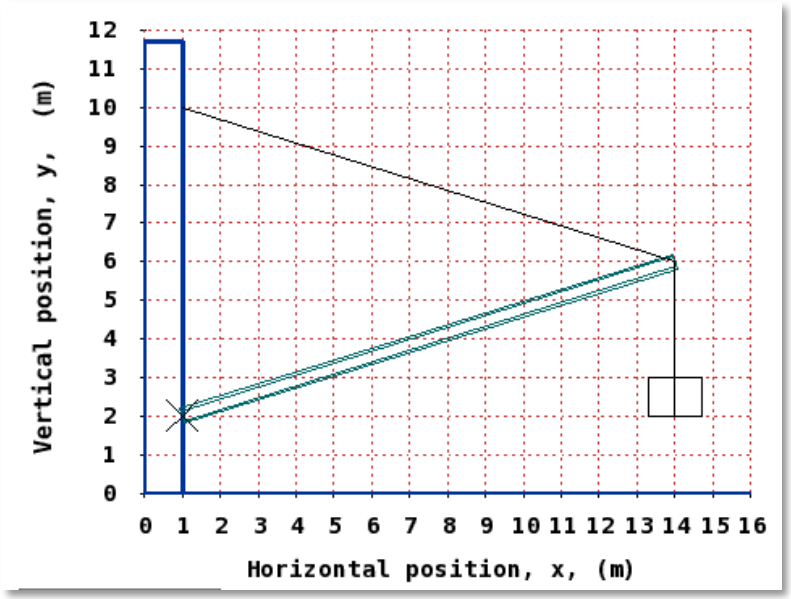
f(x)=

Submit Answer Tries 0/99

Mathematical Output

- Line graphics





Generating Mathematics Problems

- LON-CAPA problems include
  - Perl Scripting Environment
  - MAXIMA Computer Algebra System
  - R Statistics Package
- Problems not just randomized, but randomly generated with desired properties

Generating Mathematics Problems

- Direct calls to MAXIMA: $result=&cas('maxima',$expression);
- Simple example: use computer algebra system to calculate a reduced fraction

Script Delete?

```
# Construct an Egyptian Fraction that can be represented by three terms with denominators between 3 and 12
@denominators=(&random_permutation(&random(1,1000,1),(3..12)))[0..2];
$egyptian='1/'.(join('+1/',sort{$a<=>$b}(@denominators)));
$possible="A possible solution is $egyptian";

# Let the CAS figure out the value
$solution=&cas('maxima',$egyptian);
```

Write 103/165 as an Egyptian Fraction

Submit Answer Tries 0

Answer for Part: 0	**A possible solution is 1/3+1/5+1/11**

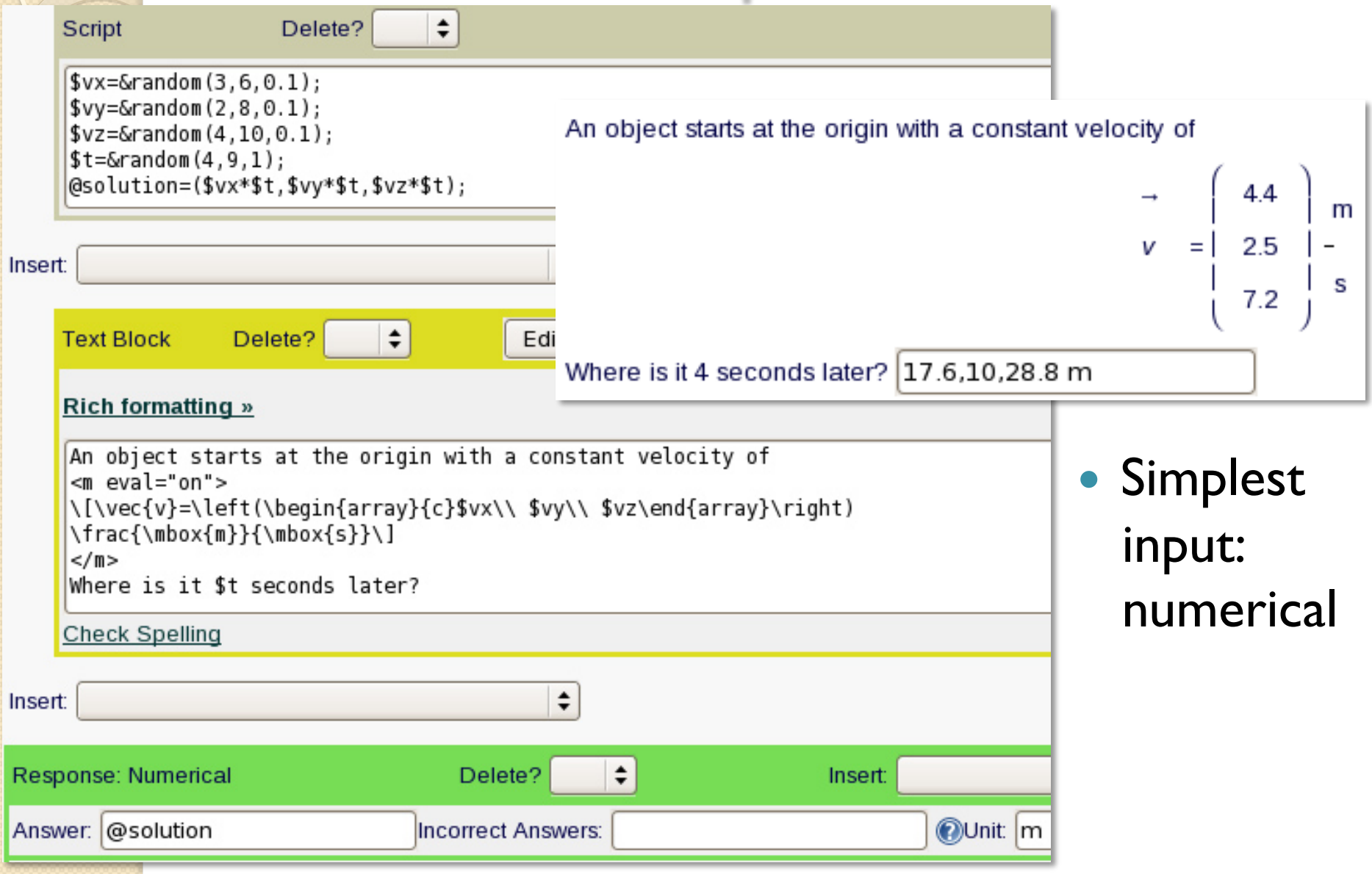
Generating Mathematics Problems

- Direct calls to R:
  $result=&cas('R',$expression);
  $results=&cas_hashref('R',$expression);
- Example: generate a distribution with certain properties:

Script Delete?

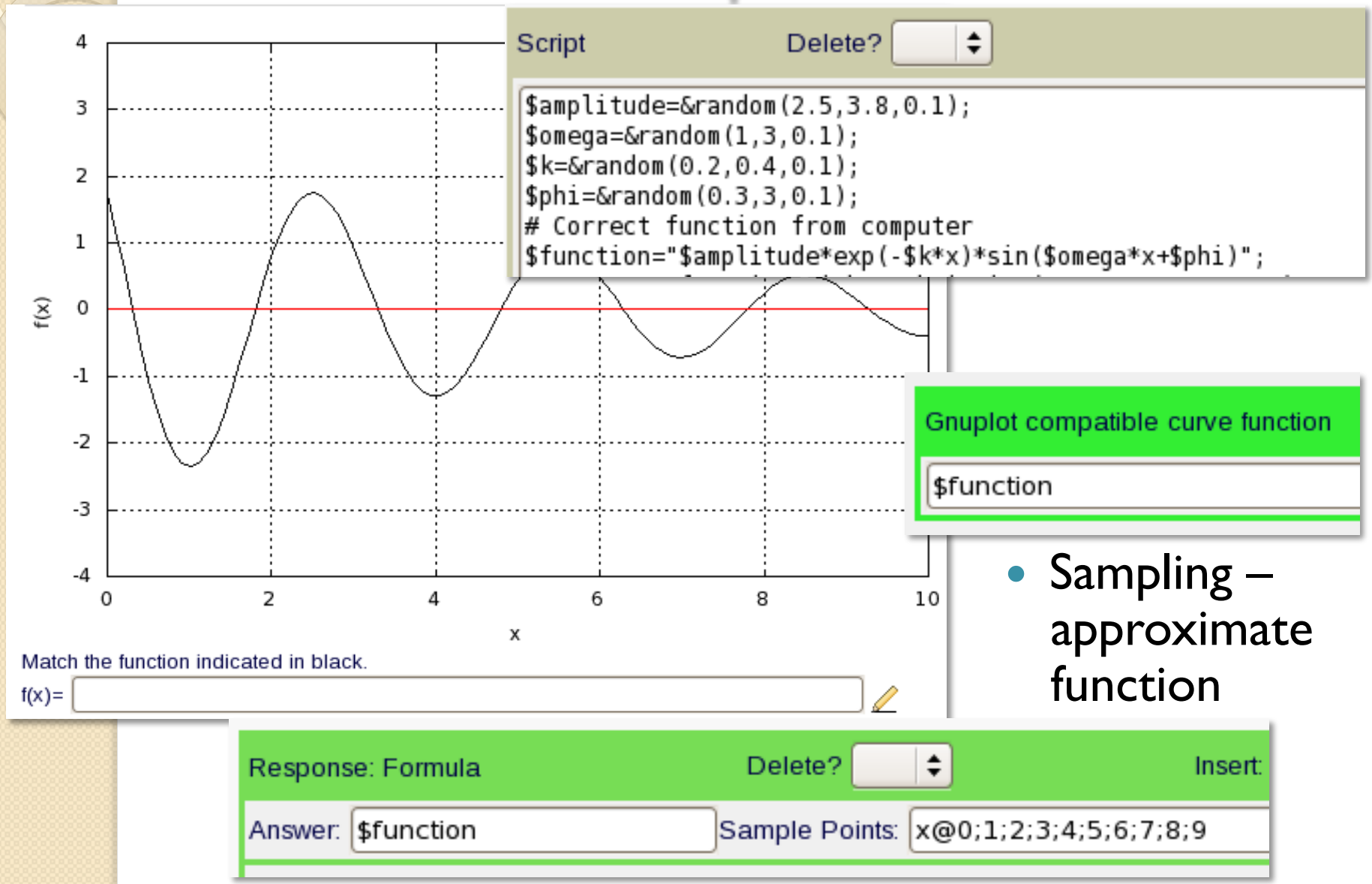
```
$seed=&random(1,500,1);
$n=&random(15,25,1);
$offset=&random(2,5,0.1);
$slope=&random(0.6,2.5,0.1);
# construct a data set using R
# dump is for debugging, print to screen to see data structure
($data,$dump)=&cas_hashref('R',"set.seed($seed);x<-1:$n;w<-1+sqrt(x)/2;data.frame(x=x,y=$offset+$slope*x+rnorm(x)*w);");
@x=&cas_hashref_array($data,'x');
@y=&cas_hashref_array($data,'y');
```

Mathematics Input



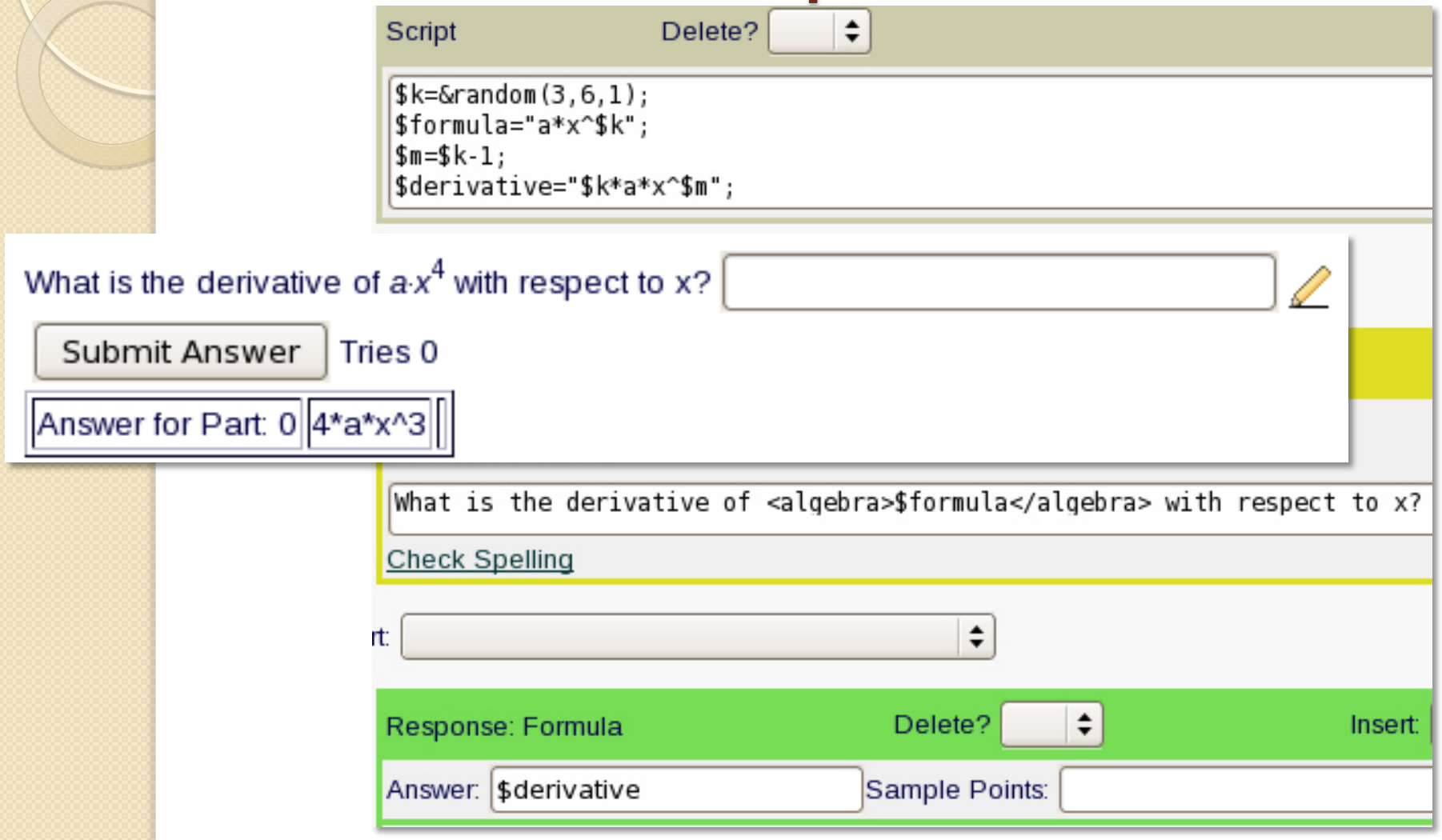
- Simplest input: numerical

Mathematics Input



- Sampling – approximate function

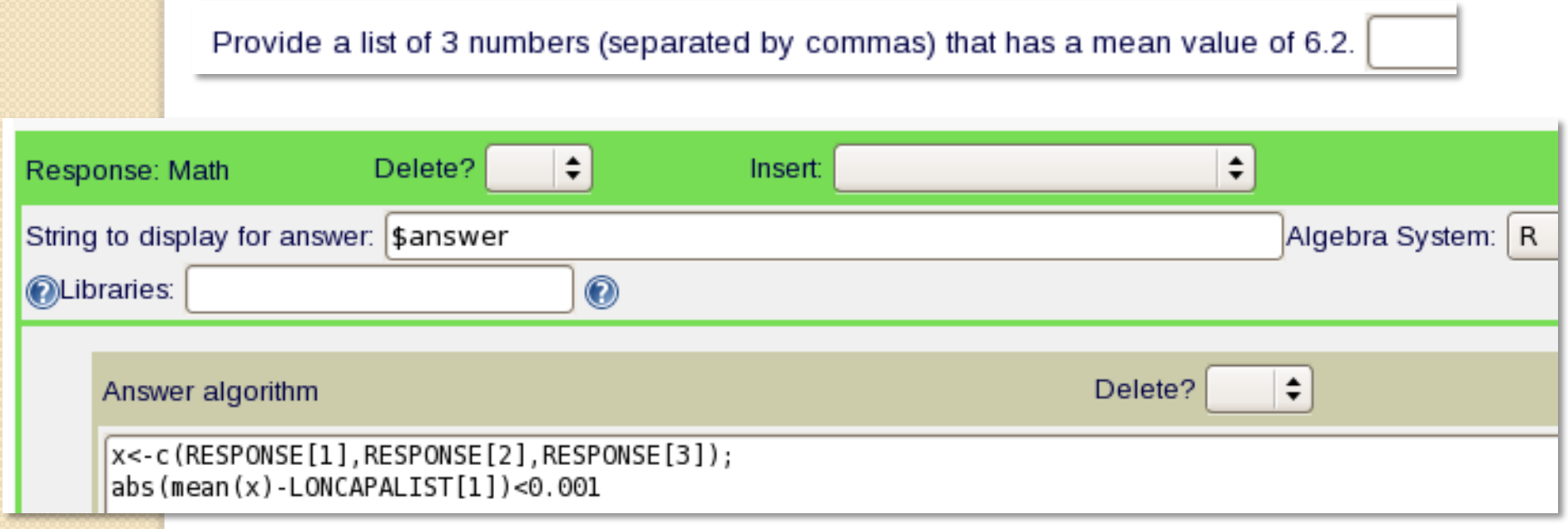
Mathematics Input



- Symbolically: exactly one exact answer (but equivalent forms)

Mathematical Input

- Checking properties
  - Using R:



Mathematical Input

- Checking properties
  - Using MAXIMA:

Give an example of a function

1. which is orthogonal to

$$-2\cdot\cos(5\cdot x)+2\cdot\sin(4\cdot x)$$

with respect to the scalar product

$$< g \mid h > = \frac{1}{\pi}\int_{-\pi}^{\pi} dx\, g(x)\cdot h(x)$$

2. whose norm is 1.

Submit Answer Tr

Answer algorithm

```
overlap:integrate((RESPONSE[1])*(LONCAPALIST[1]),x,-%pi,%pi)/%pi;
norm:integrate((RESPONSE[1])*(RESPONSE[1]),x,-%pi,%pi)/%pi;
is(overlap=0 and norm=1);
```

Mathematical Input

- Checking properties
  - Using Perl and MAXIMA:

Write 9/20 as an Egyptian Fraction

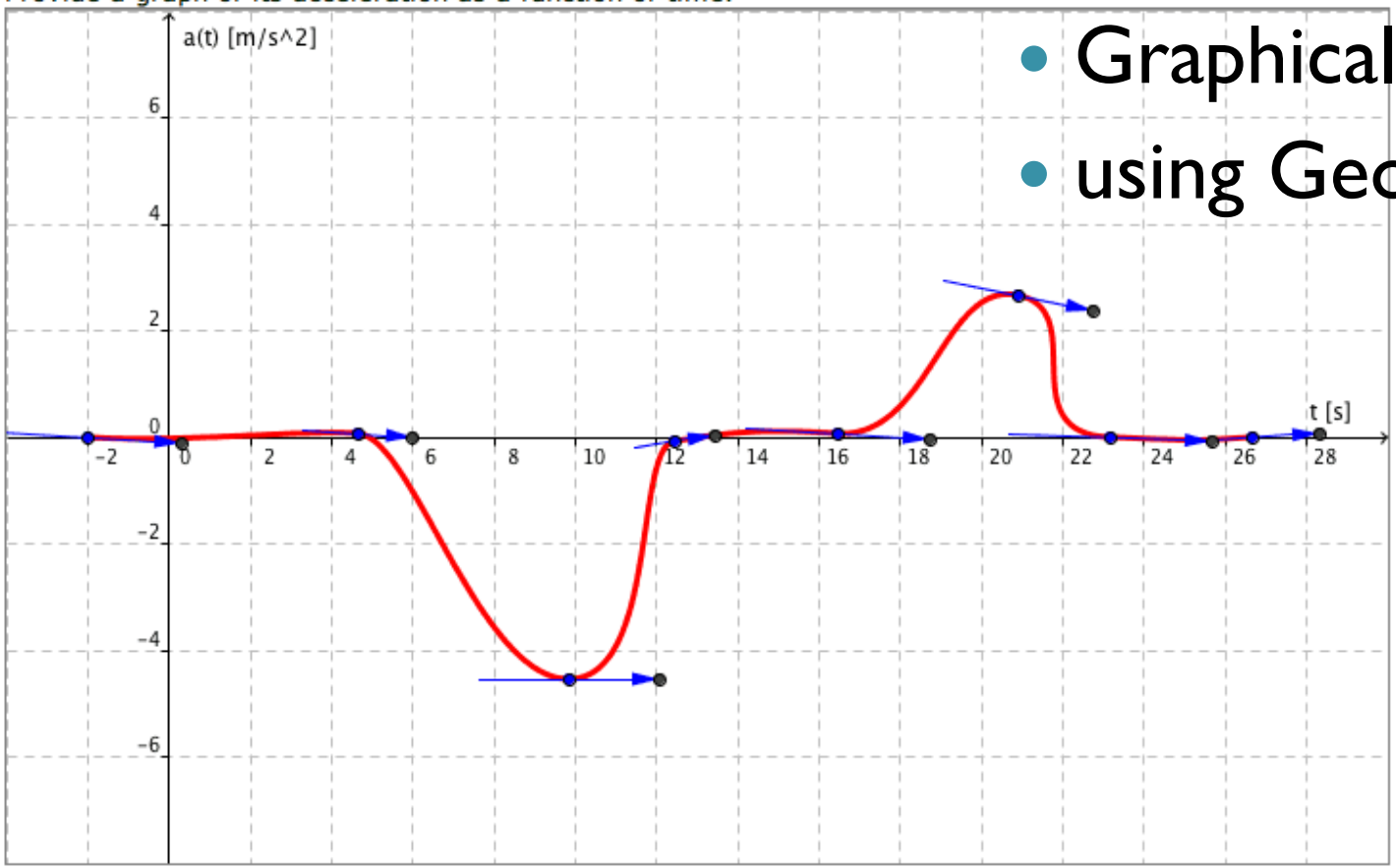
```
# Subroutine that checks if the provided term is indeed an Egyptian Fraction
sub analyze {
    my ($expression)=@_;
    $expression=~s/\s//gs;
    $expression=~s/\+?1\//\,/gs;
     if ($expression=~/^(\,[0-9]+)+$/) {
# Format is indeed 1/n+1/m+...
        $last=-1;
        foreach $number (sort { $a<=>$b } split(/\,/,$expression)) {
# Is a number used twice?
            if ($last==$number) { return(0,1); }
            $last=$number;
          }
          return(0,0);
     }
     return(1,0);
}
```

Answer algorithm Delete?

```
# Analyze the format
($formaterror,$doubleerror)=&analyze($submission);
if ($formaterror || $doubleerror) { return 'WRONG_FORMAT'; }
# It is an Egyptian Fraction, is the value correct?
if (&cas('maxima',$submission.'-('.$egyptian.')') eq '0') {
    return 'EXACT_ANS';
}
return 'INCORRECT';
```

Mathematical Input

At t=0 s, a car cruises at a constant positive velocity. Suddenly, a light switches to red. At t=10 s, the driver is maximum on the brake. The car then stops in front of the red light for over 2 seconds. Eventually, it drives off, and then again cruises at a constant velocity. The car cannot accelerate with more than 3 m/s^2.
Provide a graph of its acceleration as a function of time.



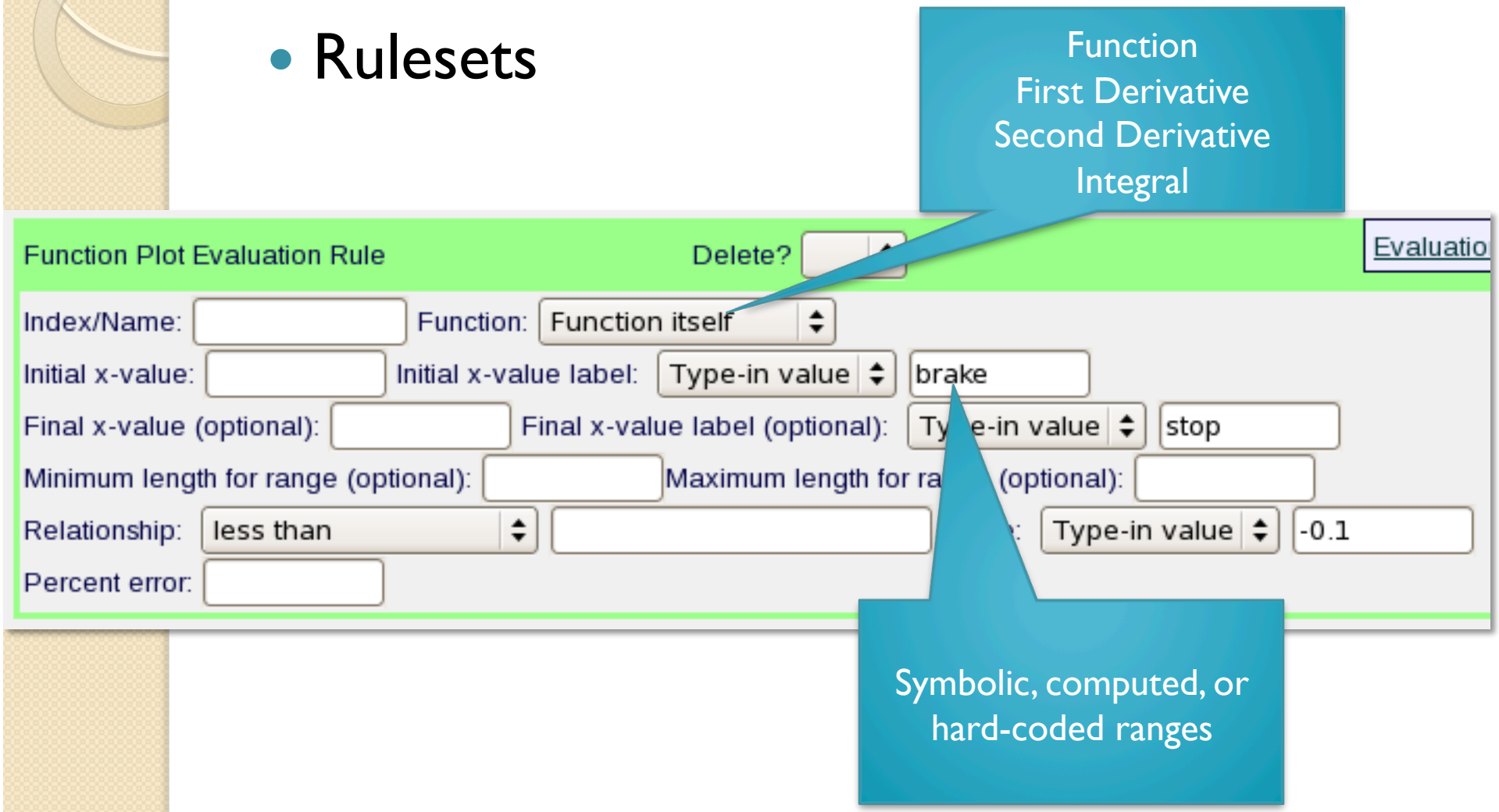
- Graphical input
- using Geogebra

You are correct. Computer's answer now shown above. Previous Tries

Script Vars
Rules Log

Mathematical Input

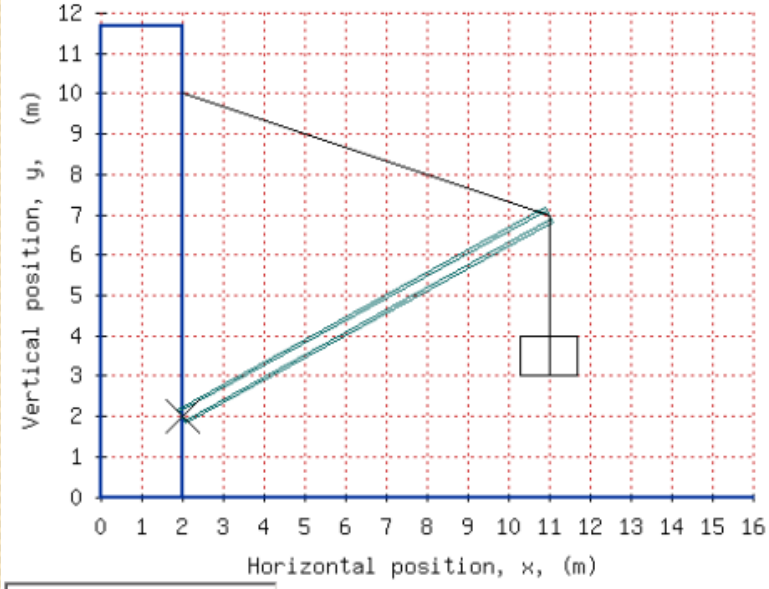
- Rulesets



Mathematical Input

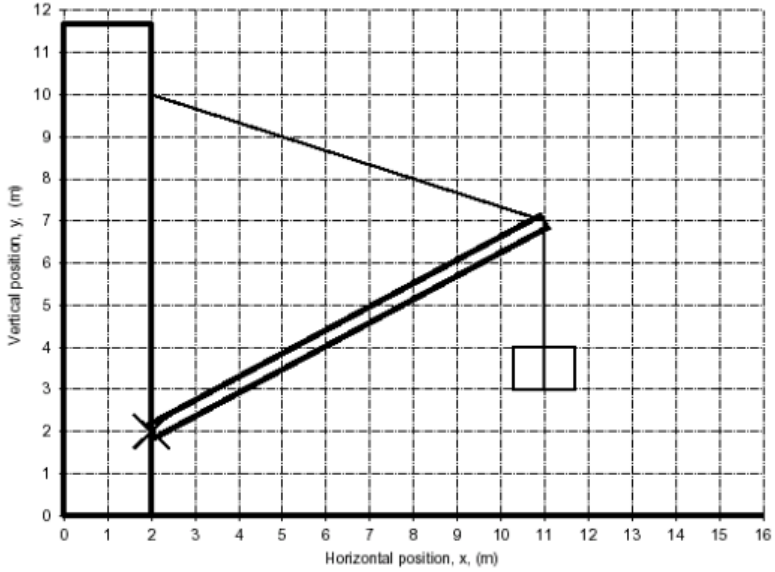
- Problems can also be rendered for bubble sheets

A crate with a mass of 177.5 kg is suspended from the end of a uniform boom with mass of 88.5 kg. The upper end of the boom is supported by a cable attached to the wall and the lower end by a pivot (marked X) on the same wall. Calculate the tension in the cable.



Submit Answer

1 pt A crate with a mass of 177.5 kg is suspended from the end of a uniform boom with mass of 88.5 kg. The upper end of the boom is supported by a cable attached to the wall and the lower end by a pivot (marked X) on the same wall. Calculate the tension in the cable.



(in N)

22. A○ 2.58×10^3 B○ 2.92×10^3 C○ 3.29×10^3
D○ 3.72×10^3 E○ 4.21×10^3 F○ 4.75×10^3
G○ 5.37×10^3 H○ 6.07×10^3

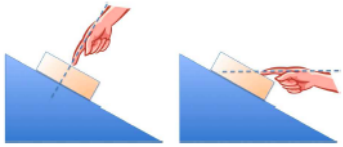
Mathematical Input

CODE - AACHDA
LB 271 - Introductory Physics Lecture
Version A

Name:

LB271 Fall 2009 Final Exam Version A

Gravitational Acceleration on Earth	$g = 9.81 m/s^2$
Gravitational Constant	$G = 6.67 \cdot 10^{-11} m^3/(kg \cdot s^2)$
Absolute Zero	-273.15°C
Gas Constant	$R = 8.31 J/(K \cdot mol)$
Boltzmann Constant	$k = 1.38 \cdot 10^{-23} J/K$
Avogadro's number	$N_A = 6.02 \cdot 10^{23}$ particles/mol
Specific heat of water vapor	$c_{vapor} = 0.48 kcal/(kg \cdot K)$
Specific heat of liquid water	$c_{water} = 1 kcal/(kg \cdot K) = 4186 J/(kg \cdot K)$
Specific heat of water ice	$c_{ice} = 0.5 kcal/(kg \cdot K)$
Latent heat of fusion for water	$L_f = 80 kcal/kg$
Latent heat of vaporizationfor water	$L_v = 540 kcal/kg$



A block is being held in place on an incline. The magnitude of the force applied by the hand on the block is the same in the left and the right scenarios.

1 pt In which scenario does the incline exert a lower normal force on the block?

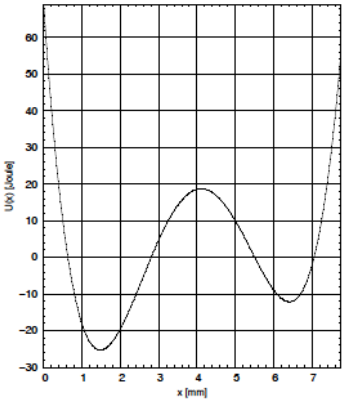
1. **A**○ The left scenario.
   B○ The right scenario.
   C○ Same in both scenarios.

1 pt In which scenario does the incline exert a lower frictional force on the block?

2. **A**○ The left scenario.
   B○ The right scenario.
   C○ Same in both scenarios.

1 pt By how many decibels does the sound intensity from a point source decrease if you increase the distance to it by a factor 6?

3. **A**○ 12.2 **B**○ 13.8 **C**○ 15.6 **D**○ 17.6
   E○ 19.9 **F**○ 22.5 **G**○ 25.4 **H**○ 28.7



1 pt
A particle is located at x=2.0 mm and has a kinetic energy of 29.5 Joule. What is the maximum x-coordinate the particle could reach? *(in mm)*

4. **A**○ 0.1 **B**○ 0.7 **C**○ 1.6 **D**○ 2.6
   E○ 3.2 **F**○ 4.7 **G**○ 5.3 **H**○ 7.6

Deep Space Nine sees Enterprise and a shuttle approach from exactly opposite directions with 0.8 c and 0.5 c, respectively.

1 pt At what fraction of the speed of light (β) does Enterprise see the shuttle approach?

5. **A**○ 0.00 **B**○ 0.50 **C**○ 0.83 **D**○ 0.91
   E○ 0.93 **F**○ 1.00 **G**○ 1.25 **H**○ 1.30

1 pt The shuttle has a length of 9 meters when at rest. How long is it in the system of Deep Space 9? *(in m)*

6. **A**○ 1.8 **B**○ 2.6 **C**○ 3.7 **D**○ 5.4
   E○ 7.8 **F**○ 11.3 **G**○ 16.4 **H**○ 23.8

1 pt Captain Picard on the Enterprise takes a 49 minute tea break. How long is this break in the system of Deep Space 9? *(in min)*

7. **A**○ 27 **B**○ 33 **C**○ 42 **D**○ 52
   E○ 65 **F**○ 82 **G**○ 102 **H**○ 128

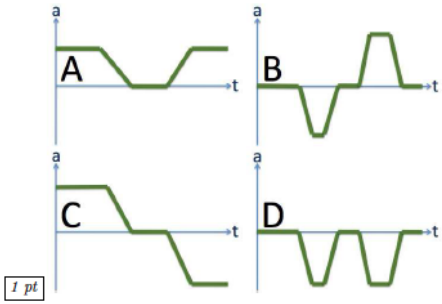
CODE - AACHDA
LB 271 - Introductory Physics Lecture
Version A

1 pt You have two organ pipes of the same length, one closed at both ends, one half open. Which one has a lower fundamental frequency?

8. **A**○ The closed pipe.
   B○ Same.
   C○ The half-open pipe.

1 pt In a very simple model of the lower atmosphere, air has a constant density of 1.26 kg/m³. How much would the air pressure change over a height difference of 130 m? *(in Pa)*

9. **A**○ 986 **B**○ 1110 **C**○ 1260 **D**○ 1420
   E○ 1610 **F**○ 1820 **G**○ 2050 **H**○ 2320

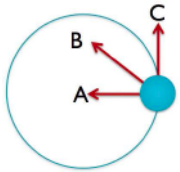


1 pt
A car drives in the forward (positive) direction. It first has a constant speed, then drives into a parking spot, waits for a few moments, and then drives out again backwards. Which one of the acceleration graphs could describe this scenario?

10. **A**○ Scenario A
    B○ Scenario B
    C○ Scenario C
    D○ Scenario D
    E○ None of the above.

1 pt
A box is sliding uphill as shown. What is the direction of the frictional force on the box?

11. **A**○ Downhill.
    B○ Perpendicular to the surface.
    C○ Uphill.
    D○ None of the above.



An object is rotating on a circular trajectory as shown. The indicated direction A is toward the center of the trajectory, C is tangential to the trajectory. The object is **rotating clockwise** and **slowing down**.

1 pt What could be the direction of the (linear) acceleration?

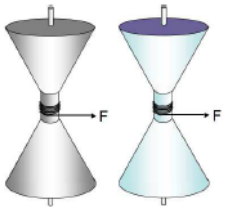
12. **A**○ Direction A.
    B○ Direction B.
    C○ Direction C.
    D○ Into the paper.
    E○ Out of the paper.

1 pt What could be the direction of the angular acceleration?

13. **A**○ Direction A.
    B○ Direction B.
    C○ Direction C.
    D○ Into the paper.
    E○ Out of the paper.

1 pt

You have two identical looking spools (same mass, same shape, same size). However, one is hollow, made from iron, the other is solid, made from aluminum. A string is wound around each spool. If you pull on both strings with equal forces, which spool is going to have the larger angular acceleration?



14. **A**○ Same
    B○ The solid spool
    C○ The hollow spool

Mathematical Input

CODE - AAFIHH
LB 271 - Introductory Physics Lecture
Version A

Name:

LB271 Fall 2009 Final Exam Version A

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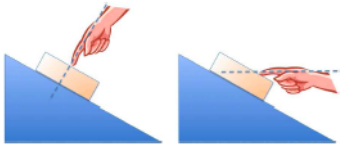
Gas Constant	$R = 8.31 J/(K \cdot mol)$
Boltzmann Constant	$k = 1.38 \cdot 10^{-23} J/K$
Avogadro's number	$N_A = 6.02 \cdot 10^{23}$ particles/mol
Specific heat of water vapor	$c_{vapor} = 0.48 kcal/(kg \cdot K)$
Specific heat of liquid water	$c_{water} = 1 kcal/(kg \cdot K) = 4186 J/(kg \cdot K)$
Specific heat of water ice	$c_{ice} = 0.5 kcal/(kg \cdot K)$
Latent heat of fusion for water	$L_f = 80 kcal/kg$
Latent heat of vaporizationfor water	$L_v = 540 kcal/kg$



A block is being held in place on an incline. The magnitude of the force applied by the hand on the block is the same in the left and the right scenarios.

1 pt In which scenario does the incline exert a higher frictional force on the block?

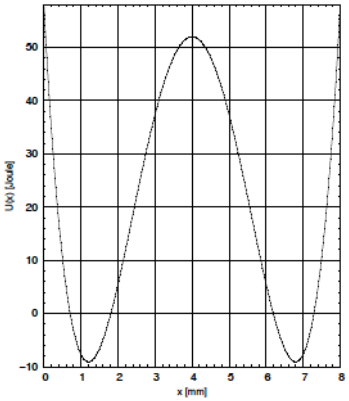
1. **A**○ The left scenario.
   B○ The right scenario.
   C○ Same in both scenarios.

1 pt In which scenario does the incline exert a higher normal force on the block?

2. **A**○ The left scenario.
   B○ The right scenario.
   C○ Same in both scenarios.

1 pt By how many decibels does the sound intensity from a point source decrease if you increase the distance to it by a factor 6?

3.A○ 7.10 **B**○ 8.31 **C**○ 9.72 **D**○ 11.4
E○ 13.3 **F**○ 15.6 **G**○ 18.2 **H**○ 21.3



1 pt
A particle is located at x=5.5 mm and has a kinetic energy of 9.8 Joule. What is the minimum x-coordinate the particle could reach? *(in* mm*)*

4.A○ 1.6 **B**○ 2.6 **C**○ 2.7 **D**○ 2.9
E○ 3.0 **F**○ 3.8 **G**○ 5.2 **H**○ 6.9

Deep Space Nine sees Enterprise and a shuttle approach from exactly opposite directions with 0.8 c and 0.4 c, respectively.

1 pt At what fraction of the speed of light (β) does Enterprise see the shuttle approach?

5.A○ 0.00 **B**○ 0.47 **C**○ 0.50 **D**○ 0.59
E○ 0.78 **F**○ 0.91 **G**○ 1.00 **H**○ 1.20

1 pt The shuttle has a length of 12 meters when at rest. How long is it in the system of Deep Space 9? *(in* m*)*

6.A○ 3.6 **B**○ 4.5 **C**○ 5.6 **D**○ 7.0
E○ 8.8 **F**○ 11.0 **G**○ 13.7 **H**○ 17.2

1 pt Captain Picard on the Enterprise takes a 35 minute tea break. How long is this break in the system of Deep Space 9? *(in* min*)*

7.A○ 19 **B**○ 28 **C**○ 40 **D**○ 58
E○ 85 **F**○ 123 **G**○ 178 **H**○ 258

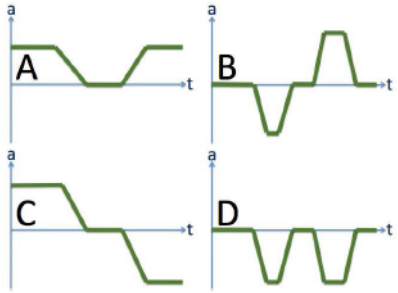
CODE - AAFIHH
LB 271 - Introductory Physics Lecture
Version A

1 pt You have two organ pipes of the same length, one closed at both ends, one half open. Which one has a lower fundamental frequency?

8. **A**○ Same.
   B○ The closed pipe.
   C○ The half-open pipe.

1 pt In a very simple model of the lower atmosphere, air has a constant density of 1.22 kg/m³. How much would the air pressure change over a height difference of 110 m? *(in* Pa*)*

9.A○ 1320 **B**○ 1490 **C**○ 1680 **D**○ 1900
E○ 2150 **F**○ 2430 **G**○ 2740 **H**○ 3100

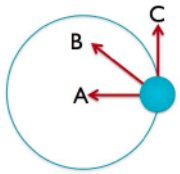


1 pt
A car drives in the forward (positive) direction. It first has a constant speed, then drives into a parking spot, waits for a few moments, and then drives out again backwards. Which one of the acceleration graphs could describe this scenario?

10. **A**○ Scenario A
    B○ Scenario B
    C○ Scenario C
    D○ Scenario D
    E○ None of the above.

1 pt
A box is sliding uphill as shown. What is the direction of the frictional force on the box?

11. **A**○ Perpendicular to the surface.
    B○ Downhill.
    C○ Uphill.
    D○ None of the above.



An object is rotating on a circular trajectory as shown. The indicated direction A is toward the center of the trajectory, C is tangential to the trajectory. The object is **rotating clockwise** and **slowing down**.

1 pt What could be the direction of the (linear) acceleration?

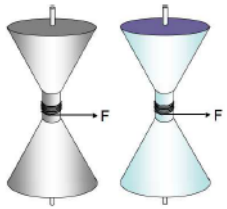
12. **A**○ Direction A.
    B○ Direction B.
    C○ Direction C.
    D○ Into the paper.
    E○ Out of the paper.

1 pt What could be the direction of the angular acceleration?

13. **A**○ Direction A.
    B○ Direction B.
    C○ Direction C.
    D○ Into the paper.
    E○ Out of the paper.

1 pt

You have two identical looking spools (same mass, same shape, same size). However, one is hollow, made from iron, the other is solid, made from aluminum. A string is wound around each spool. If you pull on both strings with equal forces, which spool is going to have the larger angular acceleration?



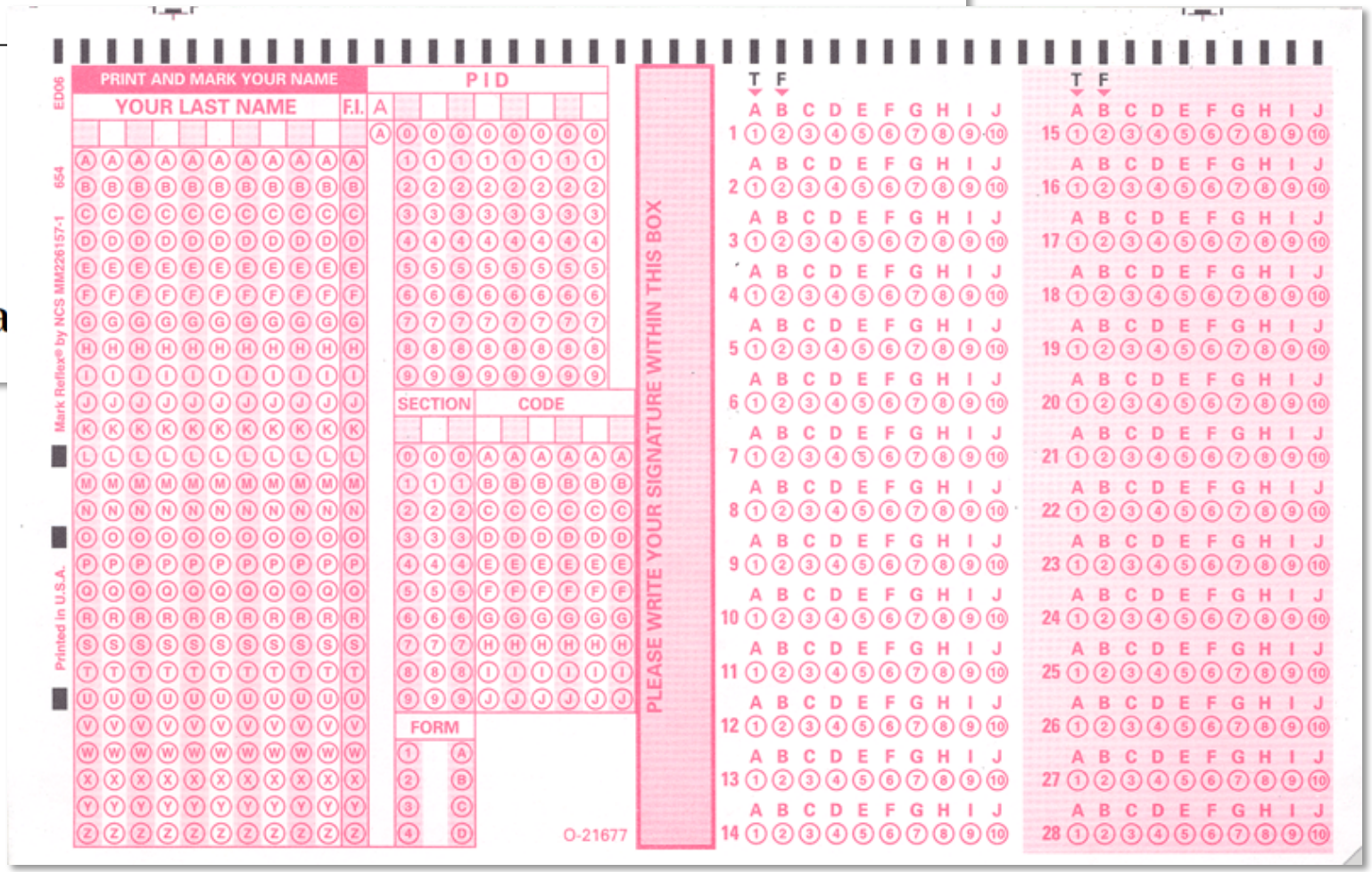
14. **A**○ The solid spool
    B○ The hollow spool
    C○ Same

Mathematical Input

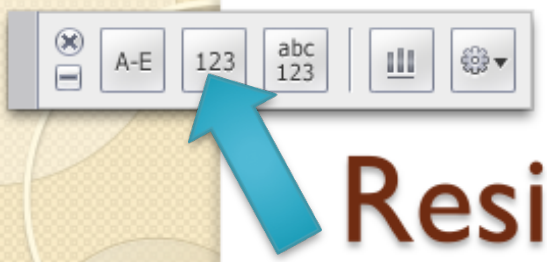
CODE - AACHDA
LB 271 - Introductory Physics Lecture
Version A

Name:

LB271 Fa



Mathematical Input



Resistance

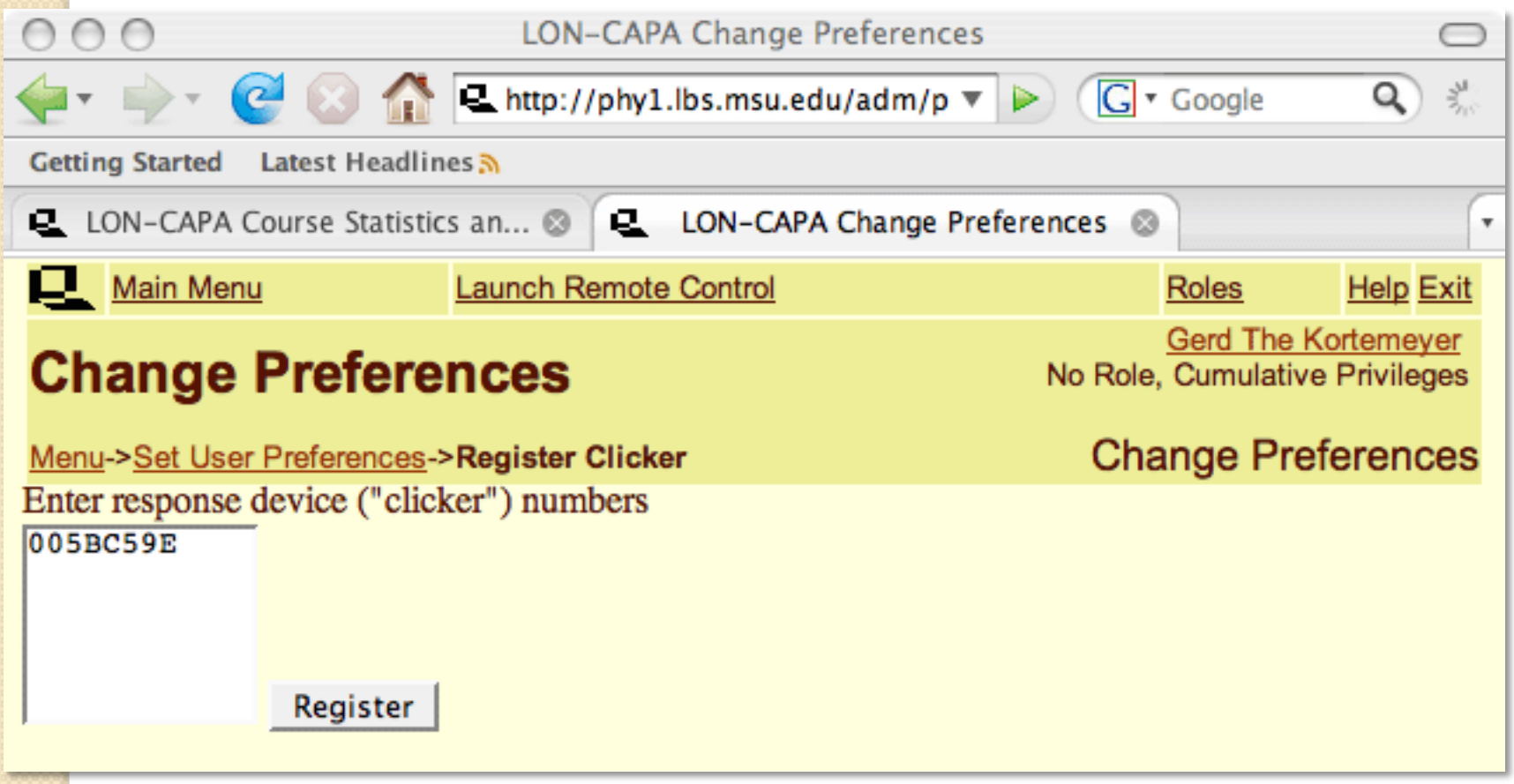
Numerical Clicker in Lecture

Current through a resistor

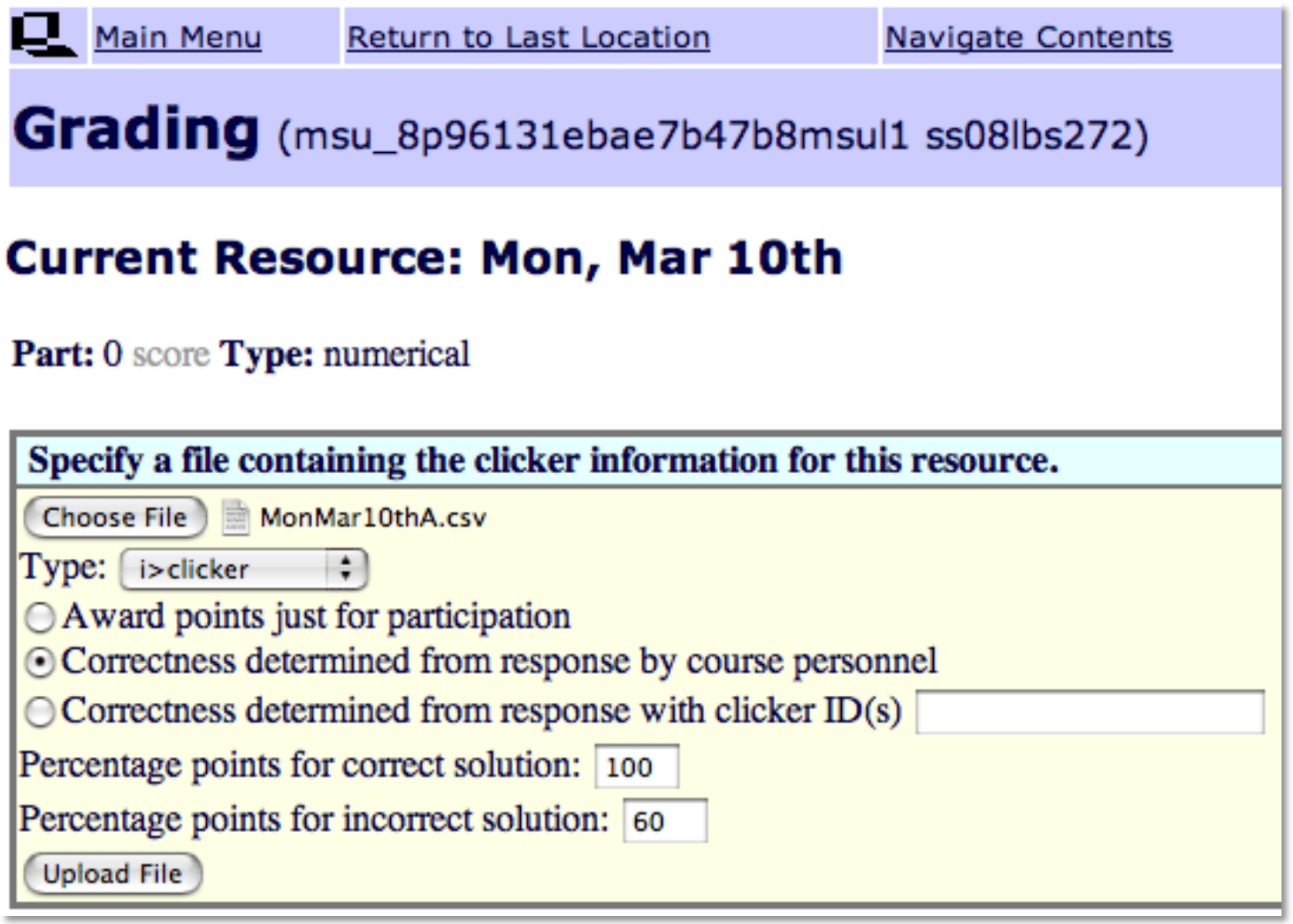
What is the current in milliampere?

Mathematical Input

LON-CAPA can evaluate clicker data after lecture

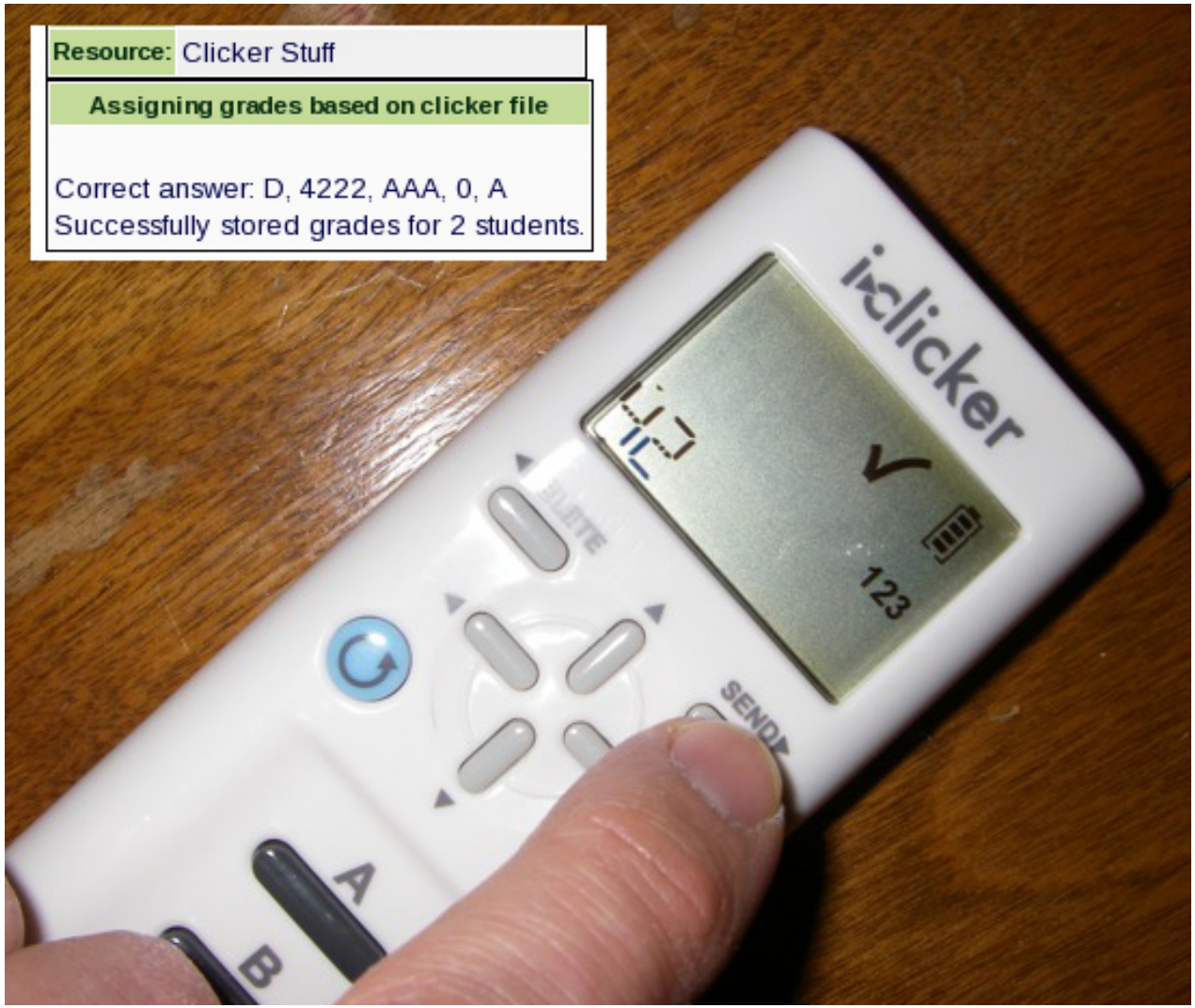


Mathematical Input



Mathematical Input

- i>clicker2 integrated in LON-CAPA



Interested?

- Faculty Seminars
  May 10-11, 2011
- Can give departmental colloquia
- Work one-on-one
- LON-CAPA Conference
  Virginia Commonwealth University
  May 19-21, 2011
- LON-CAPA Workshop
  MSU, late June

Thank You

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and

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